

# Chapter 1

## INTRODUCTION TO DIGITAL SIGNAL PROCESSING

### 1.8 Digital Filters

### 1.9 Two DSP Applications

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August 5, 2018

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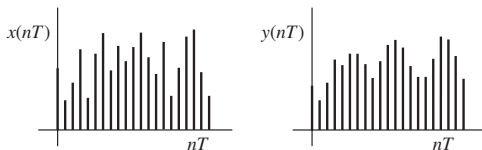
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However, it was soon realized that digital filters could also perform filtering tasks that were not even possible with analog filters.

- ◆ Since that time a great variety of digital filters have been invented such as nonrecursive, recursive, and adaptive filters.
- ◆ This presentation will examine the advantages and disadvantages of digital filters and their unique features, the types of digital filters that have evolved as well as their applications.

# Digital Filters

- ◆ In its most general form, a digital filter is a system that will receive an input in the form of a discrete-time signal and produce an output again in the form of a discrete-time signal.



- ◆ There are many types of discrete-time systems that will receive a discrete-time signal as input and produce a processed discrete-time signal as output, e.g.,
  - digital control systems,
  - data-compression systems,
  - encoders and decoders.

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  - digital control systems,
  - data-compression systems,
  - encoders and decoders.
- ◆ What differentiates digital filters from other discrete-time systems is the nature of the processing involved.

As in analog filters, the spectrum of *the output signal must be related to that of the input by some rule of correspondence.*



# Historical Background

The roots of digital filters go back in history to the 1600s when the astronomers of that time were attempting to rationalize and interpret their measurements of planetary orbits.

- ◆ The need arose in those days for a process that could be used *to interpolate a function represented by numerical data*, and a wide range of numerical interpolation formulas were proposed over the years by Gregory (1638-1675), Newton (1642-1727), Taylor (1685-1731), Stirling (1692-1770), Lagrange (1736-1813), Bessel (1784-1846).

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- ◆ On the basis of interpolation formulas, formulas were soon generated that would perform numerical differentiation or integration on a function represented by numerical data.

## Historical Background *Cont'd*

- ◆ Consider the situation where a numerical algorithm is used to compute the derivative of a signal  $x(t)$  at  $t = t_1, t_2, \dots, t_K$ , designated as  $y(t)$ , and assume that the signal is represented by its numerical values

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as output, which is a differentiated version of the input signal.

- ◆ Since differentiation is essentially a filtering process, as was demonstrated earlier on, an algorithm that performs numerical differentiation is, in fact, a digital filtering process.

## Historical Background *Cont'd*

- ◆ Interpolation formulas were put to good use during the 1600s and 1700s for the construction of numerical tables of all forms for use in business, commerce, engineering, navigation etc., for example, logarithmic and trigonometric tables.

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- ◆ In fact, it was the great need for accurate numerical tables that prompted Charles Babbage during the 1800s to embark on his lifelong quest to automate the computation process through *his famous difference engines*, and it is on the basis of numerical formulas that his machines were supposed to perform their computations.

## Historical Background *Cont'd*

- ◆ The purpose of Babbage's machines was to evaluate polynomials of the form

$$y = f(x) = a_0 + a_1x + a_2x^2 + \cdots + a_nx^n$$

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- ◆ Given a series of values of  $x$ , say,

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- ◆ In other words, a difference engine would receive a series of numbers as input and produce a series of numbers as output and, contrary to popular belief, these machines were, in effect, *discrete-time systems*.

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- ◆ More information on the roots of DSP can be found in a two-part article published by the author in: IEEE Circuits and Systems Magazine, issues no. 1 and no. 4, 2007.

Presentations based on these articles can be found at the following links:

<http://www.ece.uvic.ca/~andreas/RLectures/RootsDSP-PartI-Pres.pdf>

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- ◆ By the late 50s, a cohesive collection of techniques referred to as *data smoothing and prediction* began to emerge through the efforts of pioneers such as Blackman, Bode, Shannon, Tuckey, and others.
- ◆ During the early 60s, an entity referred to as the *digital filter* began to appear in the literature to describe a collection of algorithms that could be used for spectral analysis and data processing.

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- ◆ In 1965, Blackman authored a book titled *Data Smoothing and Prediction* and included in this work certain techniques which he referred to as *numerical filtering*.

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- ◆ From the late 60s on, the analysis and processing of signals in the form of numerical data became known as *digital signal processing*, and algorithms, computer programs, or systems that could be used for the processing of these signals *became fully established as digital filters*.

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- ◆ Because of all these important features, digital technologies can be used to design *cost-effective, reliable, and versatile systems*.

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- ◆ Hardware digital filters can be designed using a number of highly specialized interconnected VLSI chips.
- ◆ Both hardware and software digital filters can be used to process real-time or nonreal-time (recorded) signals, except that the former are usually much faster and can deal with real-time signals whose frequency spectrums extend to much higher frequencies.

- ◆ Occasionally, digital filters are used in so-called *quasireal-time applications* whereby the processing appears to a person to be in real time although, in actual fact, the samples of the signal are first collected and stored in a digital memory and are then retrieved in blocks and processed.

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- ◆ A familiar, quasireal-time application involves the transmission of radio signals over the Internet.

The signals are transmitted through data packets in a rather irregular manner, yet the music appears to be continuous only because the data packets are first stored and then properly sequenced.

This is why it takes a little while for the transmission to begin.



# Hardware Implementations

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- ◆ This means that digital filters are more suitable for applications where programmable, time-variable, or adaptive filters are required.

## Hardware Implementations *Cont'd*

- ◆ Hardware digital filters are essentially low-frequency systems where the operating frequencies are in some range, say, 0 to  $\omega_{max}$ , where  $\omega_{max}$  is dependent on the state-of-the-art in VLSI technology and the application at hand.

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- ◆ Once the sampling frequency is fixed, the sampling period  $T = 2\pi/\omega_s$  is also fixed and, consequently, a basic limitation is imposed by the amount of computation that can be performed by the digital filter during period  $T$ .

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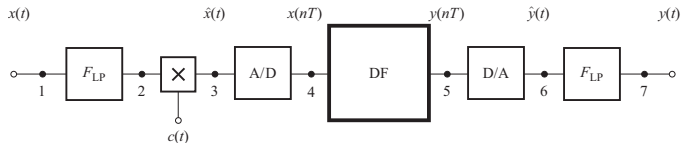
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- ◆ Eventually, at some sufficiently high sampling frequency  $\omega_{max}$ , a digital filter will become computation bound and will malfunction.
- ◆ The upper frequency of applicability,  $\omega_{max}$ , is difficult to formalize because it depends on several factors such as the number-crunching capability and speed of the VLSI chips used on the one hand, and the complexity of the filtering tasks involved on the other.



# Hardware Implementations *Cont'd*

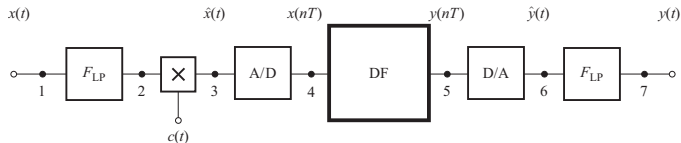
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# Hardware Implementations *Cont'd*

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- ◆ However, they have certain disadvantages as well:

For applications where the signals are of the continuous-time type, additional interfacing components are needed to make digital filters work, e.g., A/D and D/A converters.



# Comparison of Filter Technologies

The choice of filter type depends on many factors and trade-offs and is critically dependent on the type of application but the key factor is the frequency range of operation:

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- ◆ For frequencies in excess of 15 GHz, a microwave filter is the only choice.

## Comparison of Filter Technologies *Cont'd*

Type of technology	Frequency range
Digital filters	0 to $\omega_{\max}$
Discrete active <i>RC</i> filters	10 Hz to 1 MHz
Switched-capacitor filters	10 Hz to 5 MHz
Passive <i>RLC</i> filters	0.1 MHz to 0.1 GHz
Integrated active <i>RC</i> filters	0.1 MHz to 15 GHz
Microwave filters	0.5 GHz to 500 GHz

# Software Digital Implementations

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# Software Digital Implementations

- ◆ Note that *software digital filters have no counterpart in the analog world* and, therefore, for nonreal-time applications they are the only choice.
- ◆ Software digital filters find widespread applications in almost every field of science and technology.

The following two applications will illustrate what can be done:

- Processing of EKG signals
- Processing of stock exchange data

# Processing of EKG Signals

- ◆ An *electrocardiogram* (or EKG also referred to as ECG) of a healthy individual assumes a fairly well defined form although significant variations can occur from one person to the next as in fingerprints.

Yet certain telltale patterns in an EKG enable a cardiologist to diagnose certain heart conditions.

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Yet certain telltale patterns in an EKG enable a cardiologist to diagnose certain heart conditions.

- ◆ An EKG is essentially a graph representing a low-level electrical signal picked up by a pair of electrodes attached to certain well defined points on the body and connected to an electrical instrument known as the *electrocardiograph*.

## Processing of EKG Signals *Cont'd*

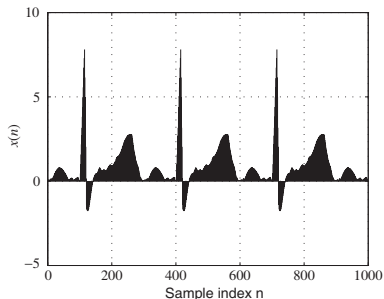
- ◆ Electrocardiographs are used in clinics and hospitals where a multitude of other types of electrical machines are utilized such as X-ray machines and electrical motors.

## Processing of EKG Signals *Cont'd*

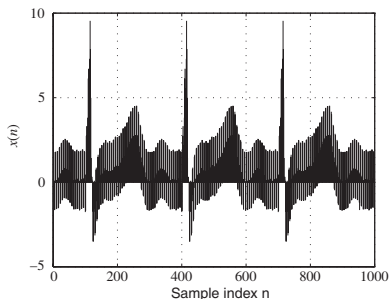
- ◆ Electrocardiographs are used in clinics and hospitals where a multitude of other types of electrical machines are utilized such as X-ray machines and electrical motors.
- ◆ All these machines along with the power lines and transformers that supply them with electricity produce electrical 60-Hz noise, which may contaminate an EKG waveform.

# Processing of EKG Signals *Cont'd*

- ◆ Fig. (a) shows a typical noise-free EKG signal and Fig. (b) shows a corresponding contaminated version.



(a) Noise-free EKG signal



(b) Contaminated EKG signal

## Processing of EKG Signals *Cont'd*

- ◆ As can be seen, the distinct features of the EKG are all but obliterated in the contaminated signal and are, therefore, difficult, if not impossible, to discern.

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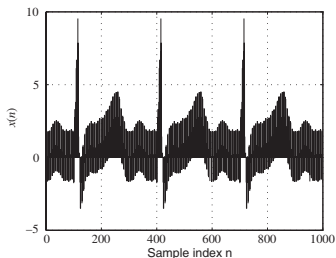
- ◆ Since electrical noise originating from the power supply has a well defined frequency, i.e., 60 Hz, one can design a bandstop filter that will reject the electrical noise.

Such a filter has been designed using the methods to be studied in later chapters and was then used to process the contaminated EKG signal.

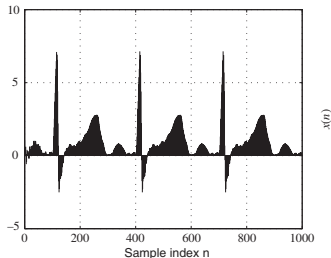


# Processing of EKG Signals *Cont'd*

- ◆ Fig. (a) shows the contaminated EKG signal and Fig. (b) shows the filtered version.



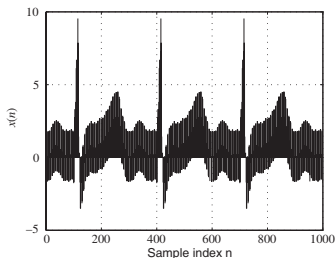
(b) Contaminated EKG signal



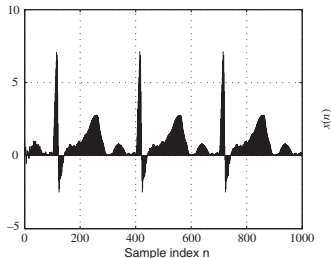
(c) Filtered contaminated EKG signal

# Processing of EKG Signals *Cont'd*

- ◆ Fig. (a) shows the contaminated EKG signal and Fig. (b) shows the filtered version.
- ◆ As can be seen, the filtered signal is a faithful reproduction of the original noise-free signal, apart from some artifacts over the interval  $n=0$  to 100 due to the transient response of the bandstop filter.



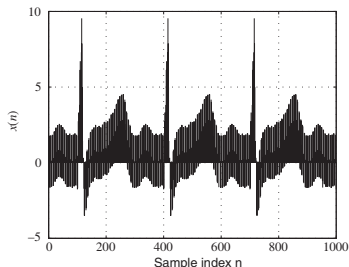
(b) Contaminated EKG signal



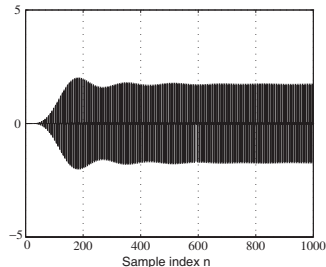
(c) Filtered contaminated EKG signal

# Processing of EKG Signals *Cont'd*

- ◆ As another experiment, just to illustrate the nature of filtering, the contaminated EKG signal was passed through a bandpass filter which was designed to select the 60-Hz noise component.



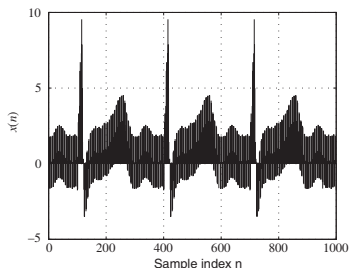
(b) Contaminated EKG signal



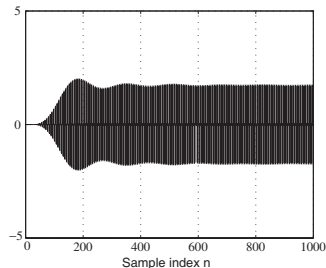
(d) Output of bandpass filter

# Processing of EKG Signals *Cont'd*

- ◆ As another experiment, just to illustrate the nature of filtering, the contaminated EKG signal was passed through a bandpass filter which was designed to select the 60-Hz noise component.
- ◆ After an initial transience over the interval  $n=0$  to 150, a steady noise component is isolated by the bandpass filter.



(b) Contaminated EKG signal



(d) Output of bandpass filter

# Processing of Stock Exchange Data

- ◆ We are all interested in the health of the market place for various reasons.

We would all like, for example, to save some money for another day and, naturally, we would prefer to invest any such funds in secure low-risk stocks, bonds, or mutual funds that provide high returns.

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- ◆ To make financial decisions such as these, we read the business section of our daily newspaper or browse the Web for numerical stock-exchange data.
- ◆ Naturally, we would like to make investments that grow steadily from year to year at a steady rate and never devalue, but this is not what happens in real life.

The prices of stocks change rapidly from one day to the next and once in a while, for example, when a market recession occurs, they can actually lose a large proportion of their values.

## Processing of Stock Exchange Data *Cont'd*

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- ◆ As long-term investors, we should perhaps ignore the day-to-day variations and focus as far as possible on the underlying changes in the stock price.
- ◆ An experienced investor may be able to draw conclusions by simply comparing the available stock-exchange data of two competing stocks.

For the rest of us this is not an easy task but through the use of DSP, the task can be greatly simplified.

## Processing of Stock Exchange Data *Cont'd*

- ◆ The price of a company's stock is a signal and, as such, it possesses a spectrum that can be manipulated through filtering.

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- ◆ If we are interested in the long-term behavior of a stock, then perhaps we should filter out the high-frequency part of the spectrum.

On the other hand, if we are interested in the volatility of the stock, then we should filter out the low-frequency content.

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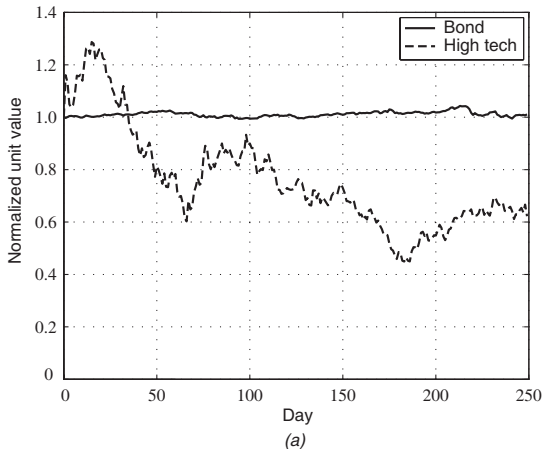
- ◆ The high-frequency or low-frequency content of a signal can be filtered out by using a lowpass or highpass filter as appropriate.

## Processing of Stock Exchange Data *Cont'd*

- ◆ To illustrate these ideas, two actual mutual funds, a bond fund and a high-tech fund, were chosen at random for processing.

One year's worth of data were chosen for processing pertaining to calendar year 2001 and to facilitate the comparison, the unit values of the two funds were normalized to unity at the start of the year, namely, January 1, 2001.

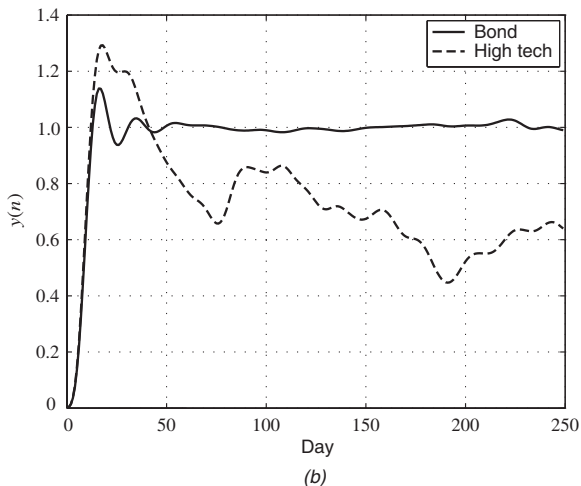
# Processing of Stock Exchange Data *Cont'd*



**Note:** 2001 was a bad year for high-technology stocks and mutual funds!



- ◆ Lowpass filtering produced the following results:



## Processing of Stock Exchange Data *Cont'd*

- ◆ The plots show certain anomalies during the first 50 or so sample values.

These are due to the initial transience that exists in all types of systems, including filters, which will be explained later on.

## Processing of Stock Exchange Data *Cont'd*

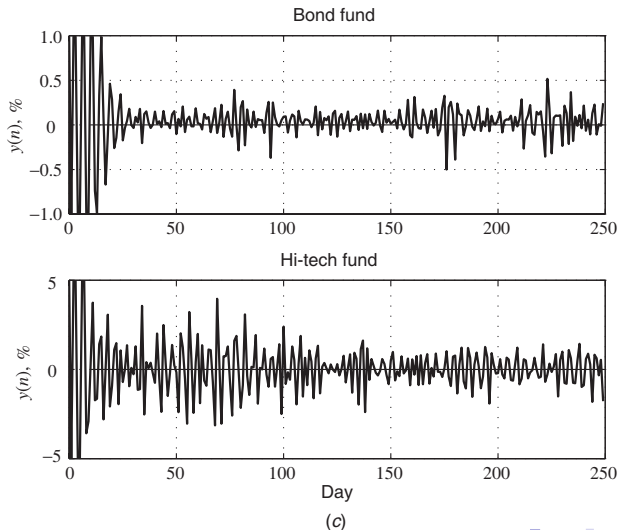
- ◆ The plots show certain anomalies during the first 50 or so sample values.

These are due to the initial transience that exists in all types of systems, including filters, which will be explained later on.

- ◆ Ignoring the initial transience, the plots show that the lowpass filter has removed the day-to-day variations and that makes it easier to discern the underlying trend of the fund.

# Processing of Stock Exchange Data *Cont'd*

- ◆ Highpass filtering produced the following results:



## Processing of Stock Exchange Data *Cont'd*

- ◆ The highpass filter removed the lowpass content but, like the lowpass filter, it introduced an initial transience.

## Processing of Stock Exchange Data *Cont'd*

- ◆ The highpass filter removed the lowpass content but, like the lowpass filter, it introduced an initial transience.
- ◆ Ignoring the initial transience and noting the difference in the y-axis scales, we observe that the amplitude of the high-frequency content in the high-tech fund is 10 times that in the bond fund.

In effect, the high-tech fund is 10 times more volatile than the bond fund, as may be expected.

*This slide concludes the presentation.  
Thank you for your attention.*